Hunter College High School Mathematics Department Re: A.P. Calculus Summer Packet

June 2015

Dear Budding Calculus Student:

Welcome to Calculus! The attached packet of exercises represents the bare minimum of mathematical prowess that you should have. **This packet is required**; you must turn it in within the first two class periods next year. Your teacher may administer a test on this material in the first two weeks.

The purpose of this summer work is to emphasize the necessity of this prerequisite knowledge for your success in Calculus. You will utilize these ideas as a launch pad for the material that will be covered. In addition to a need for solid algebra skills, your next year of math will rely heavily on functions and their notation, trigonometry, and logarithms. This packet should not be overly challenging, but if you have trouble, you should seek information on the Internet or from your previous class notes.

Given that this is an A.P. course, the pace will be fast. This makes homework completion that much more essential. If you fall behind on the homework, you will likely struggle in the course. It is also important to note that one purpose of the class is to prepare you for the Advanced Placement Test in May. It is expected that all students who take the course will also take the A.P. Test. Even after this examination, there are a few topics that the College Board assumes will be covered. These topics will round out your knowledge of Calculus and ensure that you are prepared for your experience in college mathematics the following year.

If you lose this packet, it will be posted on the math department's section of the Hunter College High School website at http://www.hchs.hunter.cuny.edu/index.php/departments/mathematics/.

Thank you and good luck! Enjoy the summer.

\*\*\*\*\* Please sign your teacher's roster \*\*\*\*\*

acknowledging that you have received the packet.

SHOW ALL WORK on a separate sheet of paper neatly and clearly, and write all answers on the answer sheet provided. Simplify.

1. 
$$\frac{x^3 - 9x}{x^2 - 7x + 12}$$
 2.  $\frac{x^3 - 8}{x - 2}$  3.  $\frac{1}{x + h} - \frac{1}{x}$  4.  $\frac{\frac{2}{x^2}}{\frac{10}{x^3}}$ 

2. 
$$\frac{x^3-8}{x-2}$$

$$3. \frac{1}{x+h} - \frac{1}{x}$$

4. 
$$\frac{\frac{2}{x^2}}{\frac{10}{x^5}}$$

5. 
$$e^{(1+\ln x)}$$

6. 
$$e^{3\ln x}$$

7. 
$$(4a^{5/3})^{3/3}$$

7. 
$$(4a^{5/3})^{3/2}$$
 8.  $(5a^{2/3})(4a^{3/2})$  9.  $\frac{3(n+1)!}{5n!}$ 

9. 
$$\frac{3(n+1)!}{5n!}$$

10. 
$$\frac{\sqrt{x}}{\sqrt[4]{x^3}} = x^a \text{ (find } a\text{)}$$

(You should be able to simplify problems 11-15 without using a calculator. Give exact answers, no decimal approximations)

12. 
$$\ln e^{7}$$

13. 
$$\log_{1/2} 8$$
 14.  $27^{2/3}$  15.  $\log 8 - \log 2$ 

Factor the expression.

16. 
$$(2x-3)^3(x+1)+(x-3)(2x-3)^2$$

Complete the following identities.

17. 
$$\sin^2 x + \cos^2 x =$$
 18.  $1 + \tan^2 x =$  19.  $\cos 2x =$ 

18. 
$$1 + \tan^2 x =$$

19. 
$$\cos 2x =$$

20. 
$$\sin 2x =$$

Solve for z in terms of x and y:

22. 
$$y^2 + 3yz - 8z - 4x = 0$$

Expand and simplify:

23. 
$$\sum_{y=0}^{4} \frac{n^2}{2}$$
 24.  $(x+y)^3$ 

Write the equation of the line:

- 25. with slope -2, containing the point (3, 4)
- 26. containing the points (1, -3) and (-5, 2)
- 27. with slope 0, containing the point (4, 2)
- 28. perpendicular to the line in problem #27, containing the point (3, 4)

Without a calculator, determine the exact value of each expression.

30. 
$$\sin \frac{\pi}{2}$$

30. 
$$\sin \frac{\pi}{2}$$
 31.  $\sin \frac{3\pi}{4}$  32.  $\cos \pi$  33.  $\cos \frac{2\pi}{3}$  34.  $\tan \frac{\pi}{2}$  35.  $\tan \frac{\pi}{6}$ 

33. 
$$\cos \frac{2\pi}{3}$$

34. 
$$\tan \frac{\pi}{2}$$

35. 
$$\tan \frac{\pi}{6}$$

For each function, determine its domain and range.

36. 
$$v = \sqrt{x-4}$$

36. 
$$y = \sqrt{x-4}$$
 37.  $y = \sqrt{x^2-4}$  38.  $y = \sqrt{x^2+4}$ 

38. 
$$y = \sqrt{x^2 + 4}$$

39. Evaluate 
$$\frac{f(x+h)-f(x)}{h}$$
 and simplify if  $f(x)=x^2-2x$ .

Determine all points of intersection.

40. 
$$y = x^2 + 3x - 4$$
 and  $y = 5x + 11$ 

40. 
$$y = x^2 + 3x - 4$$
 and  $y = 5x + 11$  41.  $y = \cos x$  and  $y = \sin x$  in the first quadrant.

Solve for x, where x is a real number. Give exact answers only, no decimal approximations.

42. 
$$\frac{x^4-1}{x^3}$$
 =

43. 
$$2x^2 + 5x = 8$$

42. 
$$\frac{x^4 - 1}{x^3} = 0$$
 43.  $2x^2 + 5x = 8$  44.  $(x+3)(x-3) > 0$  45.  $x^2 - 2x - 15 \le 0$ 

45. 
$$x^2 - 2x - 15 \le 0$$

46. 
$$12x^2 = 3x$$

47. 
$$27^{2x} = 9^{x-3}$$

46. 
$$12x^2 = 3x$$
 47.  $27^{2x} = 9^{x-3}$  48.  $\sin 2x = \sin x, 0 \le x \le 2\pi$  49.  $\log x + \log(x-3) = 1$ 

49. 
$$\log x + \log(x-3) = 1$$

50. 
$$\log_{81} x = -\frac{1}{3}$$
 51.  $\ln e^x = 4$ 

51. 
$$\ln e^x = 4$$

Graph each function: (you should be familiar with these graphs without relying on a graphing calculator)

52. 
$$y = e^{x}$$

53. 
$$y = \sqrt{x}$$

54. 
$$y = \ln x$$

55. 
$$y = \frac{1}{x}$$

56. 
$$y = \frac{1}{x^2}$$

52. 
$$y = e^x$$
 53.  $y = \sqrt{x}$  54.  $y = \ln x$  55.  $y = \frac{1}{x}$  56.  $y = \frac{1}{x^2}$  57.  $y = \begin{cases} x^2, x < 0 \\ x + 2, 0 \le x \le 3 \\ 4, x > 3 \end{cases}$ 

58. Sketch and shade the region bounded by the curves:  $y = \log_2 x$ , y = 0, x = 4, and  $y = -(x - 2)^2 + 1$ 

The last 2 questions require a graphing calculator. If you are not familiar with how to use them, you should be able to find resources online. Here is one link that might be helpful:

http://mathbits.com/mathbits/tisection/General/GraphingTidbits.htm

Round all of the following answers to the nearest thousandth (3 decimal places).

- 59. Find the coordinates of both points of intersection of  $y = \ln x$  and y = 2x 5
- 60. Find the coordinates of the maximum point of the curve  $y = \cos x (x-1)^2 + 2$  where  $-\pi \le x \le \pi$

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Enter <u>only the answers</u> on this sheet. The detailed solutions should be written on a separate sheet of paper.

1.	11.	21.
2.	12.	22.
3.	13.	23.
4.	14.	24.
5.	15.	25.
6.	16.	26.
7.	17.	27.
8.	18.	28.
9.	19.	29.
10.	20.	30.

31.	41.	51.
32.	42.	52.
33.	43.	53.
34.	44.	54.
35.	45.	55.
36.	46.	56.
37.	47.	57.
38.	48.	58.
39.	49.	59.
40.	50.	60.