SUPPLEMENTAL COMPLEX EXERCISES

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Exercise 1
Explain why $5^{\frac{3}{2}} = (\sqrt{5})^3$.

Exercise 2
Express the following complex numbers in the form $a + bi$.

(1) $\frac{1+i}{1-i}$
(2) $\frac{1}{i^5}$
(3) $(-\frac{1}{2} + i\frac{\sqrt{3}}{2})^3$
(4) $\frac{1}{(-2+i)(1-3i)}$.

Exercise 3
Compute $\sqrt{5 + 12i}$.

Exercise 4
Prove that $|z_1 \cdot z_2| = |z_1||z_2|$. Use it to show that the product of two complex numbers is zero only if one of the factors is zero.

Exercise 5
Prove that for a point $z = \cos\phi + i\sin\phi$ on the unit circle $\frac{1}{z} = \cos\phi - i\sin\phi$.

Exercise 6
Prove that $\frac{a+bi}{a-bi}$ always has absolute value 1.

Exercise 7
Interpret the angle of the complex number $\frac{(z_2-z_1)}{(z_3-z_1)}$ in the triangle formed by the vertices z_1, z_2, z_3.

Exercise 8
Prove that if for four complex numbers z_1, z_2, z_3, and z_4 the angles of $\frac{z_3-z_1}{z_3-z_2}$ and $\frac{z_4-z_1}{z_4-z_2}$ are the same, then the numbers lie on a circle or a straight line, and conversely.

Exercise 9
Find $(1 + i)^{11}$

Exercise 10
Find $\sqrt[3]{i}$

Exercise 11
Find $\sqrt[3]{7 - 4i}$

Exercise 12
Consider the complex mapping $w = f(z) = \frac{z-a}{z-b}$. Show geometrically that if we apply this mapping to the perpendicular bisector of the line segment joining a and b then the image is the unit circle.

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